1. Určeme, zda rovnice $x^2+4y^2-2x+24y+33=0$ je rovnicí elipsy v normální poloze, metodou převodu rovnice na středový tvar.

   Na levé straně rovnice doplníme členy s x a y na úplné čtverce:

$$x^2+4y^2-2x+24y+33=$$
$$=\left(x^2-2x+1\right)-1+4\left(y^2+6y+9\right)-4\cdot 9+33=$$
$$=\left(x-1\right)^2+4\left(y+3\right)^2-4=0.$$

   Výslednou rovnici lze snadno upravit na rovnici elipsy v normální poloze:

$$\left(x-1\right)^2+4\left(y+3\right)^2=4\Rightarrow \underline{\underline{\frac{\left(x-1\right)^2}{2^2}+\frac{\left(y+3\right)^2}{1^2}=1.}}$$

2. Jaká je středová rovnice elipsy, je-li $a=3; b=5; S=[0;0]$

$$\frac{x^2}{b^2}+\frac{y^2}{a^2}=1 \qquad \frac{x^2}{5^2}+\frac{y^2}{3^2}=1 \qquad \frac{x^2}{25}+\frac{y^2}{9}=1$$

3. Jaká je obecná rovnice elipsy $\left(x+\sqrt{2}\right)^2+\frac{\left(y-\sqrt{2}\right)^2}{4}=1$

$$\left(x+\sqrt{2}\right)^2+\frac{\left(y-\sqrt{2}\right)^2}{4}=1 \quad |\cdot 4$$
$$4\left(x+\sqrt{2}\right)^2+\frac{4\left(y-\sqrt{2}\right)^2}{4}=4$$
$$4\left(x+\sqrt{2}\right)^2+\left(y-\sqrt{2}\right)^2=4$$

$$4\left(x^2+2\sqrt{2}x+2\right)+\left(y^2-2\sqrt{2}y+2\right)=4$$
$$4x^2+8\sqrt{2}x+8+y^2-2\sqrt{2}y+2-4=0$$
$$4x^2+y^2+8\sqrt{2}x-2\sqrt{2}y+8+2-4=0$$

$$4x^2+y^2+8\sqrt{2}x-2\sqrt{2}y+6=0$$

4. Jaké jsou středová rovnice elipsy $\underline{4x^2+9y^2-8x-36y+4=0}$, velikost a, b, e a souřadnice S, F_1, F_2, A, B, C a D?

$$4x^2-8x+9y^2-36y+4=0$$
$$4\left(x^2-2x\right)+9\left(y^2-4y\right)+4=0$$
$$4\left(x^2-2x+1\right)-1\cdot 4+9\left(y^2-4y+4\right)-4\cdot 9+4=0$$
$$4\left(x-1\right)^2+9\left(y-2\right)^2-4-36+4=0$$
$$4\left(x-1\right)^2+9\left(y-2\right)^2=36 \quad |:36$$
$$\frac{4\left(x-1\right)^2}{36}+\frac{9\left(y-2\right)^2}{36}=\frac{36}{36}$$
$$\frac{\left(x-1\right)^2}{9}+\frac{\left(y-2\right)^2}{4}=1$$

$$S=[m;n]=[1;2]$$
$$a^2=9\Rightarrow a=3$$
$$b^2=4\Rightarrow b=2$$
$$a^2=b^2+e^2\Rightarrow e=\sqrt{a^2-b^2}=$$
$$=\sqrt{9-4}=\sqrt{5}$$

$$F_1=[m-e;n]=\left[1-\sqrt{5};2\right]$$
$$F_2=[m+e;n]=\left[1+\sqrt{5};2\right]$$
$$A=[m-a;n]=[-2;2]$$
$$B=[m+a;n]=[4;2]$$
$$C=[m;n+b]=[1;4]$$
$$D=[m;n-b]=[1;0]$$